No Confinement without Coulomb Confinement

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We compare the physical potential $V_D(R)$ of an external quark-antiquark pair in the representation D of SU(N), to the color-Coulomb potential $V_{\rm coul}(R)$ which is the instantaneous part of the 44-component: of the gluon propagator in Coulomb-gauge $D_{44}(\vec{x}, t) = V_{\rm coul}(|\vec{x}|)\delta(t) +$ (noninstantaneous). We show that if $V_D(R)$ is confining, $\lim_{R\to\infty} V_D(R) = +\infty$, as is believed to hold in the absence of dynamical quarks, then the inequality $V_D(R) \leq -C_D V_{\rm coul}(R)$ holds asymptotically at large R, where $C_D > 0$ is the Casimir in the representation D. This implies that $-V_{\rm coul}(R)$ is also confining.

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Introduction.—The problem of confinement of color charge has been with us for a long time. There are many approaches to this problem such as dual Meissner effect by monopole condensation [1], effective string theory [2], center dominance [3], relegation of colored states to the unphysical, indefinite-metric space [4], and color-Coulomb potential [5]. One seeks to choose variables so that the most important degrees of freedom have a simple expression. For this purpose gauge fixing can be a useful technique.

Confinement is most commonly characterized by the behavior of $V_D(R)$, the gauge-invariant potential energy between an external quark-pair at separation R in the representation D of the gauge structure group SU(N). It may found from a rectangular Wilson loop $W_D(R,T)$ in the representation D, but for our purposes it is more convenient to obtain it from the correlator, $\langle P_D(\vec{x})P^*_D(\vec{y})\rangle$, of a pair of Polyakov or thermal Wilson loops at $\vec{x}$ and $\vec{y}$ in the representation D, on a Euclidean lattice of period T in the 4-direction. The Polyakov loop is the lattice analog of the continuum expression $P_D(\vec{x}) = {\rm tr}[P\exp(\int_0^T A_{D,4}(\vec{x},t)dt]$, where $A_{D,\mu} \equiv A^a_\mu t^a_D$, and the t^a_D satisfy the Lie algebra commutation relations $[t^a_D, t^b_D] = f^{abc}t^c_D$ in the representation D. As discussed recently [2], this correlator has the expansion

$$\langle P_D(\vec{x})P^*_D(\vec{y})\rangle = \sum_{n=0}^{\infty} \exp(-E_{n,\vec{x},\vec{y}}T), \qquad (1)$$

where the $E_{n,\vec{x},\vec{y}}$ are the eigenvalues, $H\Psi_n = E_{n,\vec{x},\vec{y}}\Psi_n$, of the lattice QCD Hamiltonian H, specified below, that includes an external quark and antiquark at $\vec{x}$ and $\vec{y}$ in the representation D. In the large-T limit, the sum is dominated by the first term, with lowest energy eigenvalue, $E_{0,\vec{x},\vec{y}}$. It is rotationally symmetric in the continuum limit, and we identify the physical quark-antiquark potential $V_D(R)$ with this energy eigenvalue, after separation of divergences,

$$E_{0,|\vec{x}_1-\vec{x}_2|} = E_0 + \Delta + V_D(|\vec{x} - \vec{y}|). \qquad (2)$$

Here E_0 is the energy of the vacuum state in the absence of an external quark-pair, and Δ is the diverging self-energy of the external quarks, namely, twice the energy obtained from a single external quark represented by a single Polyakov loop $\langle P_D(\vec{x})\rangle$. According to the Wilson confinement criterion [6], which is expected to hold in pure gluodynamics without dynamical quarks, but with external quarks in the fundamental representation, $V_F(R)$ diverges linearly at large R, $V_F(R) \sim \sigma R$ where σ is the conventional string tension. As Wilson himself pointed out, in an Abelian theory the Wilson loop is given by $\exp[-g^2 \oint dx^\mu \oint dy^\mu D_{\mu\nu}(x-y)]$, where $D_{\mu\nu}(x-y)$ is the free gluon propagator. Although no such result holds in a non-Abelian theory, a suggestive earlier result in Coulomb gauge [5] was that, if one keeps only the contributions to the Wilson loop that come from the instantaneous part $V_{\rm coul}(R)$ of the gluon propagator, then these form a set of ladder diagrams that sums to $\exp[C_D T V_{\rm coul}(R)]$. However, there was no estimate of the remainder. According to the result that will be established here, this quantity provides a bound on the Wilson loop, $\exp[C_D T V_{\rm coul}(R)] \leq \exp[-TV_D(R)]$, that holds in the confining case, asymptotically at large T and R, with $T \gg R$.

In the Coulomb gauge, there is a simple scenario [5] that attributes confinement of color charge to the long range of the color-Coulomb potential, $V_{\rm coul}(R)$. This quantity characterizes the instantaneous part of the 44-component of the gluon propagator $\langle A^a_4(\vec{x},t)A^b_4(0,0)\rangle = D_{44}(\vec{x},t)\delta^{ab} = V_{\rm coul}(|\vec{x}|)\delta^{ab} +$ (noninstantaneous). Since A_4 couples universally to the color charge, this can account for confinement of color charge, provided that $V_{\rm coul}(R)$ is indeed long range. A remarkable feature of the Coulomb gauge in QCD, a property not shared by any Lorentz gauge, is that $A_4 = g_0 A^{(0)}_4 = g_r A^{(r)}_4$ is a renormalization-group invariant [5,7]. Here g_0 and $A^{(0)}_4$, and g_r and $A^{(r)}_4$ are, respectively, the unrenormalized and renormalized charges and perturbative gauge connections. This means that D_{44}, and hence also its instantaneous part $V_{\rm coul}(R)$, is independent of both the cutoff Λ and the renormalization mass μ. This property allows the fundamental QCD quantity, $V_{\rm coul}(R)$, the instantaneous part of the gluon propagator, to be identified with the phenomenological QCD potential [8,9]. Its Fourier transform, $\tilde{V}_{\rm coul}(|\vec{k}|)$, provides a convenient definition of the running coupling constant, $\alpha_s(|\vec{k}|/\Lambda_{\rm coul}) = g^2_{\rm coul}(|\vec{k}|)/(4\pi) = \vec{k}^2\tilde{V}_{\rm coul}(|\vec{k}|)/(4\pi x_0)$ where $x_0 = 12N/(11N - N_f)$, $\Lambda_{\rm coul}$ is a finite QCD mass scale, and N_f is the number of quark flavors [10]. The result obtained here means that if the Wilson criterion for confinement is satisfied, then, with this definition, the running coupling constant $\alpha_s(|\vec{k}|/\Lambda_{\rm coul})$ diverges in the infrared like $1/\vec{k}^2$, a clear manifestation of infrared slavery.

We shall show that a necessary condition for confinement according to the Wilson criterion is that the instantaneous color-Coulomb potential be confining. In symbols: if $\lim_{R\to\infty} V_D(R) = +\infty$, then $V_D(R) \leq -C_D V_{\rm coul}(R)$ holds asymptotically at large R. (The minus sign occurs because antiquark has opposite charge to quark.) Here $C_D = -\sum_a (t^a_D)^2 > 0$ is the (positive) value of the Casimir invariant, and in the fundamental representation $C_F = (N^2 - 1)/2N$. Notable economy is hereby achieved because $V_D(R)$ is defined by means of a Wilson loop, which is a path-ordered exponential in the gluon field A, and is thus an infinite sum over gluon n-point functions of all orders n, whereas $V_{\rm coul}(R)$ is defined in terms of the gluon 2-point function, D_{44}. By contrast, in Landau gauge the gluon propagator is short range, and the confinement mechanism is less obvious, requiring perhaps an infinite sum of diagrams. We also note the striking numerical result [11,12] that $V_D(R)$ exhibits Casimir scaling, $V_D(R) = (C_D/C_F)V_F(R)$ quite accurately at least in a rather large range of separation R, and eight different representations D. This suggests that the above bound may be saturated in this range, for this would explain Casimir scaling, that is not easy to understand otherwise [13]. The result also makes it imperative to extend present programs to calculate $V_{\rm coul}(R)$ numerically [14], and analytically from first principles [9], and to derive phenomenological quantities from it [8].

Lattice Coulomb-gauge QCD Hamiltonian.—The energy $E_{0,|\vec{x}_1-\vec{x}_2|}$ is of course gauge invariant, and the lattice QCD Hamiltonian H may be chosen in any gauge. Its most familiar form is in the temporal gauge $U_4 = 1$, corresponding to $A_4 = 0$,

$$H_{\rm temp} = g_0^2(2a)^{-1}\sum_{\vec{x},1}\varepsilon^2_{\vec{x},i} + 2(g_0^2 a)^{-1}\sum_p {\rm Re\,Tr}U_p, \quad (3)$$

where $\sum_p$ is the sum over all spatial plaquettes p (on a single time-slice). Here $\varepsilon_{\vec{x},i}$ is the color-electric field operator that satisfies $[\varepsilon^a_{\vec{x},i}, U_{\vec{y},j}] = it^a U_{\vec{y},j}\delta_{\vec{x},\vec{y}}\delta_{i,j}$ and $[\varepsilon^a_{\vec{x},i}, \varepsilon^b_{\vec{y},j}] = -if^{abc}\delta_{\vec{x},\vec{y}}\delta_{i,j}\varepsilon^c_{\vec{x},i}$. We place an external quark at $\vec{x}_1$ in the representation D, and an external antiquark at $\vec{x}_2$ in the representation D^*, with color vectors that act on the first and second indices of the wave functional $\Psi_{\alpha\beta}(U)$ according to $(\lambda^a_1\Psi)_{\alpha\beta} = (\lambda^a_D)_{\alpha\gamma}\Psi_{\gamma\beta}$ and $(\lambda^a_2\Psi)_{\alpha\beta} = -(\lambda^a_D)^*_{\beta\gamma}\Psi_{\alpha\gamma}$, where $\lambda^a_D = it^a_D$. In the temporal gauge, the color charges of the external quarks do not appear in the Hamiltonian $H_{\rm temp}$, but rather in the subsidiary condition $G^a(\vec{x})\Psi = 0$. This is an expression of Gauss's law, for $G^a(\vec{x})$ is a precise lattice analog of the continuum Gauss's law operator $G^a(\vec{x}) = -(\vec{D}\cdot\vec{E})^a(\vec{x}) + \rho^a_{qu}(\vec{x})$, where $E^a_i(\vec{x}) = i\frac{\delta}{\delta A^a_i(\vec{x})}$, $D^{ac}_i = \delta^{ac}\partial_i + f^{abc}A^b_i$ is the gauge-covariant derivative, and $\rho^a_{\rm qu}(\vec{x}) = \lambda^a_1\delta(\vec{x}-\vec{x}_1) + \lambda^a_2\delta(\vec{x}-\vec{x}_2)$ is the color-charge density of the external quarks. In the temporal gauge $G^a(\vec{x})$ is the generator of three-dimensionally local gauge transformations of the quark and gluon variables, satisfying $[G^a(\vec{x}), G^b(\vec{y})] = i\delta(\vec{x}-\vec{y})f^{abc}G^c(\vec{x})$, and the subsidiary condition is the statement of gauge invariance of the wave functional.

One would expect that Gauss's law is essential for confinement, and the lattice Coulomb Hamiltonian $H_{\rm coul}$ [15] may be derived from $H_{\rm temp}$ by solving Gauss's law as subsidiary condition [16]. For our purposes the resulting lattice Coulomb Hamiltonian has the same structure as the continuum Coulomb Hamiltonian [17]. To simplify the exposition, we shall use continuum language, but it is understood that this is shorthand for the correct lattice kinematics, and divergences are controlled by use of the lattice Coulomb Hamiltonian, as will be made clear.

To get to the Coulomb gauge from the temporal gauge, one integrates out the gauge degrees of freedom using the Faddeev-Popov formula in all gauge-invariant matrix elements. In particular, for the Hamiltonian, one obtains $H_{\rm coul}$ defined by its matrix elements

$$(\Psi_1, H_{\rm coul}\Psi_2) = \int_\Lambda dA^{\rm tr}{\rm det}M(1/2) \times \int d^3x[g_0^2(E^a_i\Psi_1)^*E^a_i\Psi_2 + g_0^{-2}\Psi_1^*\vec{B}^2\Psi_2], \quad (4)$$

where the wave functionals $\Psi_{\alpha\beta}(A^{\rm tr})$ depend only on three dimensionally transverse continuum configurations, $\partial_i A^{\rm tr}_i = 0$, a contraction on color indices is understood, and det M is the determinent of the three-dimensional Faddeev-Popov operator, $M^{ac}(A^{\rm tr}) \equiv -D^{ac}_i(A^{\rm tr})\partial_i = -\partial_i D^{ac}_i(A^{\rm tr}) = -\partial^2\delta^{ac} - f^{abc}A^{{\rm tr},b}_i\partial_i$. The color-magnetic field is given by $B^a_1 = \partial_2 A^{{\rm tr},a}_3 - \partial_3 A^{{\rm tr},a}_2 + f^{abc}A^{{\rm tr},b}_2 A^{{\rm tr},c}_3$, etc., and the color-electric field by $E^a_i = E^{{\rm tr},a}_i - \partial_i\phi^a$, where $E^{{\rm tr},a}_i = i(\delta/\delta A^{{\rm tr},a}_i)$, and $\phi^a(\vec{x})$ is the color-Coulomb potential operator. In this matrix element, $\phi^a(\vec{x})$ acts directly on the wave functional Ψ. The definition of $H_{\rm coul}$ is completed by specifying that $\phi^a(\vec{x})\Psi \equiv (M^{-1}\rho_{\rm phys})^a(\vec{x})\Psi$, where $\rho^a_{\rm phys} \equiv -f^{abc}A^{{\rm tr},b}_i E^{{\rm tr},c}_i + \rho^a_{\rm qu}$ is the sum of the color-charge density of the external quarks $\rho^a_{\rm qu}$, defined above, plus the color charge of the dynamical gluon degrees of freedom only. This is the solution of the subsidiary condition $G^a(\vec{x})\Psi = 0$, or $M^{ac}(A^{\rm tr})\phi^c\Psi = \rho^a_{\rm phys}\Psi$ and expresses $\phi^a(\vec{x})\Psi$ in terms of $\rho_{\rm qu}$ and the transverse gluon variables. The charge,

$Q^a = \int d^3x \rho^a_{\rm phys}(\vec{x})$ may be identified with the physical color charge, for it generates global gauge tramformatiom on all variables $[Q^a, A_i^{{\rm tr},b}] = if^{abc}A_i^{{\rm tr},c}$ etc., and satisfies $[Q^a, Q^b] = if^{abc}Q^c$. The second term in M is characteristic of non-Abelian gauge theory. It is responsible for antiscreening because, for typical configurations, this term produces a small denominator in M^{-1}. The subscript Λ on the integral $\int_\Lambda dA^{\rm tr}$ means that a region that includes only one Gribov copy is integrated over. This may be chosen as in the minimal Coulomb gauge, but the proof does not depend on the particular way this is chosen. Note that when the solution of the subsidiary condition is substituted into the Hamiltonian, the dependence on the color-charge vectors λ_1 and λ_2 and positions $\vec{x}_1$ and $\vec{x}_2$ of the external quark pair has moved from the subsidiary condition (that has been eliminated) into the Hamiltonian $H_{\rm coul}$, although we have not yet written this dependence explicitly.

Bound on $V_D(R)$ from trial wave function.—The energy $E_{|\vec{x}_1-\vec{x}_2|} \equiv (\Psi, H_{\rm coul}\Psi)$ of any trial wave function Ψ provides an upper bound on the ground-state energy, $E_{0,|\vec{x}_1-\vec{x}_2|} \leq E_{|\vec{x}_1-\vec{x}_2|}$. As trial function we take the product wave function, $\Psi_{\alpha\beta}(A^{\rm tr}) = N_D^{-1/2}\delta_{\alpha\beta}\Phi_0(A^{\rm tr})$. Here $\Phi_0(A^{\rm tr})$ is the exact wave functional of the vacuum state in the absence of external quarks, and $N_D^{-1/2}\delta_{\alpha\beta}$, where N_D is the dimension of representation D, is the external quark-pair state of total color-charge zero, $(\lambda_1 + \lambda_2)^a\Psi = 0$. The Coulomb Hamiltonian has the decomposition $H_{\rm coul} = H_{\rm gl} + H_{\rm gl,qu} + H_{\rm qu,qu}$, that follows from the decomposition of the color-charge density $\rho^a_{\rm phys} = -f^{abc}A_i^{{\rm tr},b}E_i^{{\rm tr},c} + \rho^a_{\rm qu}$ in $\phi\Psi \equiv M^{-1}\rho_{\rm phys}\Psi$, where $\rho^a_{\rm qu}$ is defined above. Here $H_{\rm gl}$ is the Coulomb Hamiltonian in the absence of external quarks, $H_{\rm gl,qu}$ is linear in $\rho_{\rm qu}$, and $H_{\rm qu,qu} = (1/2)\int d^3x(\partial_i M^{-1}\rho_{\rm qu})^2(\vec{x})$. There is no ordering problem in the last expression because $\rho_{\rm qu}$ commutes with $A^{\rm tr}$. By definition of Φ_0, we have $H_{\rm gl}\Phi_0 = E_0\Phi_0$, where E_0 is the vacuum energy in the absence of external quarks. We also note the further decomposition $H_{\rm qu,qu} = H_{1,1} + H_{2,2} + H_{1,2} + H_{2,1}$, that follows from the decomposition $\rho^a_{\rm qu}(\vec{x}) = \lambda_1^a\delta(\vec{x}-\vec{x}_1) + \lambda_2^a\delta(\vec{x}-\vec{x}_2)$. Thus from $(\Psi, H_{\rm gl}\Psi) = E_0$, and $(\Psi, H_{\rm gl,qu}\Psi) = 0$, we get for the trial energy,

$$E_{|\vec{x}_1-\vec{x}_2|} = E_0 + \Delta'(\Lambda) - C_D U_{\rm coul}(|\vec{x}_1-\vec{x}_2|, \Lambda), \qquad (5)$$

where we have used $(N^2-1)\langle\lambda_1^a\lambda_2^b\rangle = \delta^{ab}\langle\lambda_1^c\lambda_2^c\rangle = -\delta^{ab}\langle\lambda_1^c\lambda_2^c\rangle = -\delta^{ab}C_D$. Here $\Delta'(\Lambda) \equiv 2(\Phi_0, H_{1,1}\Phi_0)^{\rm latt}$ is (another) self-energy of the external quarks that, by translation invariance of Φ_0, is independent of $\vec{x}_1$ and $\vec{x}_2$, where the superscript on the matrix element is a reminder that the corresponding lattice expression is understood. Likewise $-C_D U_{\rm coul}(|\vec{x}_1-\vec{x}_2|, \Lambda) \equiv 2(\Phi_0, H_{1,2}\Phi_0)^{\rm latt}$, is the interaction energy of the two external quarks. It is given by $U_{\rm coul}(|\vec{x}_1-\vec{x}_2|, \Lambda)\delta^{ab} \equiv (\Phi_0, [M^{-1}(-\partial^2)M^{-1}]^{a}_{\vec{x}_1,\vec{x}_2}{}^{b}\Phi_0)^{\rm latt}$ which we call the lattice color-Coulomb potential. Both quantities depend on the ultraviolet cutoff $\Lambda = a^{-1}$, where a is the lattice spacing. The inequality $E_{0,|\vec{x}_1-\vec{x}_2|} \leq E_{|\vec{x}_1\to\vec{x}_2|}$ reads $\Delta(\Lambda) + V_D(R,\Lambda) \leq \Delta'(\Lambda) - C_D U_{\rm coul}(R,\Lambda)$. We have canceled the vacuum energy E_0 that diverges with the volume of space and, having done so, we may take the volume of space to infinity, keeping the ultraviolet cutoff in place.

Having separated out the self-energies, both the lattice quark potential $V_D(R,\Lambda)$ and the lattice color-Coulomb potential $U_{\rm coul}(R,\Lambda)$ have finite, Λ-independent continuum limits. For on the one hand we have $\lim_{\Lambda\to\infty} V_D(R,\Lambda) = V_D(R)$ where $V_D(R)$ is the finite physical potential energy between a pair of external quarks. On the other hand in the continuum limit, the above matrix element is, remarkably, none other than the instantaneous part, of the 44-component of the gluon propagator $D_{44}(\vec{x}, t)$ introduced in the Introduction, $V_{\rm coul}(|\vec{x}_1-\vec{x}_2|)\delta^{ab} = (\Phi_0, [M^{-1}(-\partial^2)M^{-1}]^{a}_{\vec{x}_1,\vec{x}_2}{}^{b}\,\Phi_0)$. This is established in Eqs. (25), (27), and (28) of [10]. Moreover, as shown in [5,7], $V_{\rm coul}(R)$ is a renormalization-group invariant, and thus independent of the cutoff Λ, and we obtain $\lim_{\Lambda\to\infty} U_{\rm coul}(R,\Lambda) = V_{\rm coul}(R)$. If $V_D(R,\Lambda)$ is confining, $\lim_{R\to\infty} V_D(R,\Lambda) = +\infty$, then, for sufficiently large R, the self-energies $\Delta(\Lambda)$ and $\Delta'(\Lambda)$ are negligible compared to $V_D(R,\Lambda)$, and the inequality $V_D(R,\Lambda) \leq -C_D U_{\rm coul}(R,\Lambda)$ holds asymptotically at large R, for finite cutoff Λ. This bound also holds in the continuum limit, because dimensional and renormalization-group considerations tell us that the terms that vanish as $\Lambda \to \infty$ are of relative order $1/(\Lambda R)^n$, where n is positive, so they also vanish asymptotically at large R. [This condition is necessary, as shown by the following counterexample: Take $V_D(R,\Lambda) = \sigma R$, and $U_{\rm coul}(R,\Lambda) = c/R + m^4R^2/\Lambda$, with self-energies $\Delta(\Lambda) = a\Lambda$, and $\Delta'(\Lambda) = (a+1)\Lambda$. The inequality at finite Λ reads $\sigma R \leq \Lambda + c/R + m^4R^2/\Lambda = 2m^2R + c/R + (\Lambda - m^2R)^2/\Lambda$. It is satisfied for all finite Λ and R, provided that $\sigma < 2m^2$ and $c \geq 0$. But the continuum limit of $U_{\rm coul}(R,\Lambda)$ is c/R, and $R < c/R$ does not hold at large R.] We conclude that if $V_D(R)$ is confining, $\lim_{R\to\infty} V_D(R) = +\infty$, then in the continuum limit, the inequality, $V_D(R) \leq -C_D V_{\rm coul}(R)$ holds asymptotically at large R, as asserted.

Conclusion.—The bound implies that if the potential between external quarks in the fundamental representation increases linearly at large R, $V_F(R) \sim \sigma R$, where σ is the standard string tension, then the color-Coulomb potential $V_{\rm coul}(R)$ increases at least linearly at large R, and moreover if its increase is also linear, $-V_{\rm coul}(R) \sim \sigma_{\rm coul}R$, as has been conjectured [5], where $\sigma_{\rm coul}$ is a string tension that characterizes $V_{\rm coul}(R)$, then the conventional string tension satisfies the bound $\sigma \leq (N^2-1)/(2N)\sigma_{\rm coul}$.

What has been learned about QCD dynamics? We have found that if the Wilson confinement criterion holds, then $V_{\rm coul}(R)$, the instantaneous part of the gluon propagator D_{44} in Coulomb gauge, is confining. Moreover, from $V_{\rm coul}(|\vec{x}-\vec{y}|) = \langle[M^{-1}(-\partial^2)M^{-1}]_{\vec{x},\vec{y}}\rangle$, this can happen only if the three-dimensional Faddeev-Popov or ghost Green function $[M^{-1}(A^{\rm tr})]_{\vec{x},\vec{y}}$ is long range for configurations $A^{\rm tr}$ that dominate the functional integral. This confirms the confinement scenario originally proposed by Gribov [18], and advocated by the author [5]. [The scenario reads, in brief, that in the minimal Coulomb gauge, configurations are restricted to the Gribov region, where the Faddeev-Popov operator is positive, $M(A^{\rm tr}) > 0$. The boundary of this region occurs where the lowest eigenvalue of $M(A^{\rm tr})$ vanishes, $\lambda_0(A^{\rm tr}) = 0$. Moreover the dimension n of configuration space is very large, being of the order of the volume V of the lattice. Entropy favors a population highly concentrated close to this boundary—where $\lambda_0(A^{\rm tr})$ is small—for the same reason that, in a space of very high dimension n, the density of a ball $r < r_0$ is very sharply peaked near r_0, being given by $r^{n-1}dr$. Consequently, for the configurations that dominate the functional integral, $M(A^{\rm tr})$ is enhanced, and thus also $V_{\rm coul}(R)$. At the same time, the would-be physical three-dimensionally transverse components of the gluon propagator are suppressed in the infrared.] As in the confinement theory of Nishijima [4], the *unphysical* degrees of freedom play an essential role in confining color, and we note the presence of the ghost propagator in the long-range instantaneous color-Coulomb potential.

To simplify the exposition, we considered gluodynamics without dynamical quarks, but they may be included as follows. Leaving aside the separate issue of the choice lattice quark Hamiltonian $H_{\rm quark}$, we note that, given such a Hamiltonian, the total Coulomb-gauge lattice Hamiltonian is $H_{\rm coul} + H_{\rm quark}$. Here $H_{\rm coul}$ is as above, except that color-charge density $\rho_{\rm phys}$ now includes a contribution from the dynamical quarks that is the lattice analog of $q^\dagger \lambda^a q$, where q is the quark spinor field. The above proof at the level of the lattice inequality that holds at all separation R (both finite and asymptotic) goes through as before. However, if dynamical quarks are present in the fundamental representation F, as occurs in nature, we are unable to derive an inequality that holds in the continuum limit, for in this case, the physical potential energy $V_{\rm F}(R)$ between external quarks does not diverge with R. For at some radius R_b the string breaks by polarization of sea quarks from the vacuum, and for $R > R_b$, $V_F(R)$ represents a residual potential between a pair of mesons, analogous to the van der Waals potential. In this case the bound obtained here does not imply that $V_{\rm coul}(R)$ is confining. Nevertheless, according to the confinemerit scenario in Coulomb gauge, $V_{\rm coul}(R)$ is a fundamental quantity that remains linearly rising even when $V_F(R)$ is not. It is precisely the linear rise of $V_{\rm coul}(R)$ that causes string breaking, by making it energetically preferable to polarize sea quarks from the vacuum.

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